การสำรวจมโนทัศน์ที่คลาดเคลื่อนและทักษะกระบวนการทางคณิตศาสตร์ก่อนเรียน ของนักเรียนชั้นมัธยมศึกษาปีที่ 6 ในการแก้สมการและอสมการ

Investigation Grade 12 Students' Prior Misconceptions and Mathematical Process Skills in Solving Linear Equation and Inequalities

กมลทิพย์ องอาจยุทธนากร
ศิลปศาสตรมหาบัณฑิต สาขาวิชาคณิตศาสตร์ศึกษา วิทยาลัยนานาชาติ มหาวิทยาลัยราชภัฏสวนสุนันทา
ที่ปรึกษาวิทยานิพนธ์ : รองศาสตราจารย์ฉวีวรรณ แก้วไทรฮะ
สาขาวิชาคณิตศาสตร์ศึกษา วิทยาลัยนานาชาติ มหาวิทยาลัยราชภัฏสวนสุนันทา

Abstract

The purpose of this study was to investigate students' prior misconceptions and errors in mathematical process skills. The method used was a quantitative research with descriptive statistics. The research instrument was a set of quizzes including solving linear equations and inequalities. The data collections were done by using 31 Grade 12 students' quiz scores. The results were analyzed and showed that students' prior misconceptions and errors in mathematical process skills to solve linear equation: using substitution (16.13%), elimination (29.03%), and graphing methods (48.39%) and solve linear inequalities: using rules of inequalities (54.84%) and graphing (58.06%). This study was a research line for designing the real classroom action research on solve linear programming problems using infographics to improve students' achievement and mathematical skills.

Keywords : linear equation and inequalities, linear programming, infographics, misconception, mathematical process skill

บทคัดย่อ

จุดมุ่งหมายของการทำวิจัยในครั้งนี้คือ เพื่อสำรวจมโนทัศน์ที่คลาดเคลื่อนหรือข้อผิดพลาดของนักเรียนในทักษะกระบวนการทางคณิตศาสตร์ก่อนเรียน วิธีการที่ใช้เป็นการวิจัยเชิงปริมาณที่อธิบายด้วยสถิติเชิงพรรณนา เครื่องมือในการทำวิจัยเป็นข้อสอบที่เกี่ยวกับการแก้สมการและอสมการ ข้อมูลทั้งหมดจะถูกรวบรวมจากคะแนนสอบของนักเรียนชั้นมัธยมศึกษาปีที่ 6 จำนวน 31 คน การวิเคราะห์ผลคะแนนแสดงให้เห็นว่า มโนทัศน์ที่คลาดเคลื่อนหรือข้อผิดพลาดของนักเรียนในทักษะกระบวนการทางคณิตศาสตร์ก่อนเรียนในการแก้สมการ : ด้วยวิธีการแทนค่า (16.13%) วิธีการกำจัด (29.03%) และวิธีการเขียนกราฟ (48.39%) และแก้อสมการ : ด้วยการใช้กฎของอสมการ (54.84%) และวิธีการเขียนกราฟ (58.06%) การศึกษาครั้งนี้ เพื่อเป็นแนวทางในการออกแบบการวิจัยในชั้นเรียนในการแก้ปัญหากำหนดการเชิงเส้นด้วยการใช้อินโฟกราฟิกเพื่อพัฒนาผลสัมฤทธิ์ทางการเรียนและทักษะทางคณิตศาสตร์ของนักเรียน

คำสำคัญ : สมการและอสมการ กำหนดการเชิงเส้น อินโฟกราฟิก มโนทัศน์ที่คลาดเคลื่อน ทักษะกระบวนการทางคณิตศาสตร์

Statement and significance of the problem

Improving students' learning mathematical process skills helps them succeed in many mathematics lessons. The mathematical process skill is an essential part of mathematics curriculum in the basic education core curriculum B.E. 2551 (A.D. 2008). Through mathematical process skills in learning mathematics help the students to build meaning and performance for a wide range of connection, representation and reasoning skills.

The present study was aimed to investigate the students' prior misconceptions and errors in mathematical process skills to learn equations and inequalities. This study was one part of pre-practicum at one practicum school in Bangkok. The researcher gathered data in particular on the misconceptions and errors relative to mathematical process skills in solving linear equation and inequalities. This study could be at least help the researcher to design the framework of real classroom action research on solve linear programming problems using infographics to improve students' achievement and mathematical skills for Grade 12 students in Semester 1, Academic Year 2018 at the practicum school.

Literature and Theory

Various studies (Hailikari, et al., 2008; Svinicki, 1993; Ambrose, et al., 2010, p.10-39; Campbell & Campbell, 2009; National Research Council, 2000) have indicated that students' prior knowledge plays the important role in students' academic success. They must connect the new knowledge into their existing knowledge, construct new concepts, and revise current idea in mastering new content.

Prior Knowledge for Solving Linear Equations and Inequalities

According to the Basic Education Core Curriculum 2008 (mathematics), solving linear equations and inequalities is a major focus in learning 'Linear Programming'. Students should be developed an understanding of the rules and processes to solve equations and inequalities. In this study, the students study two methods to find the solution to equations and inequalities with two variables by using 'Algebraically Method' and 'Graphically Method' as shown in Table 1.

Table 1 : Methods to Solve Equations and Inequalities with Prior Knowledge

Methods	Prior Knowledge
1. Solving Linear Equation	
1.1 Algebraically Method - Substitution - Elimination 1.2 Graphically Method	1. The rules behind numerical computations--number and the order of operations, the properties of operations, and the properties of equality; 2. The letters as representing specific unknown values, as in $3x - 4y = 8$; 3. Plotting points in Cartesian Plane.
2. Solving Inequalities	
2.1 Algebraically Method 2.2 Graphically Method	1. The rules behind numerical computations--number and the order of operations, the properties of operations, and the properties of inequalities; 2. The letters as representing specific unknown values, as in $3x - 4y \geq 8$ or $3x - 4y \leq 8$; 3. Plotting points in Cartesian Plane.

Source : Project Maths Development Team (2012); Ministry of Education. (2008)

Mathematical Process Skills

There are five process standards are required to encourage students learn mathematics-1)) Problem Solving, (2) Reasoning and Proof, (3) Communication, (4) Connections, and (5) Representations. Principles of mathematical communication refer to ways of sharing ideas to others orally or in writing. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language (NCTM, 2000). In addition, the role of

communication including writing can enhance the learning of mathematics and extend student's thinking and understanding. Providing students an opportunity to develop their thinking as well as to communicate in mathematics can deepen students understanding in mathematics (Mayo, 2007; Pugalee, 2005). In learning mathematics, students need to be encouraged to use mathematical communication and multiple representations including mathematics words, symbols, graphical and numerical form of mathematics concepts. The process of communication develops students with appropriate mathematics language, builds understanding of concepts learned, and helps students to reason mathematically.

The development of students' mathematical communication skills is specific to mathematical background knowledge and the ability to express their ideas clearly when speaking or writing about mathematics so that others can understand the concepts and process in mathematics (Sammons, 2018).

Throughout the first three weeks of school, the researchers observed mathematics classroom for pre-practicum while high school students were learning with an experienced mathematics teacher of the school. When students were working mathematics exercises in the textbook, they were struggling with a concept and did mistakes in solving linear equations. This topic is a background knowledge of learning the next topic on 'Linear Programming'.

After this observation, the researchers decided to find the new strategies to help students learn more confident in mathematics and complete the exercise with ease. Students have different styles in learning mathematics such as Using infographic in teaching mathematics is one strategy which combined the information (content) and graphic to enhance the students' learning of mathematics with respect to elements of the visual graphics.

Infographics for Mathematics

Infographics are visual representations of information, data, or knowledge in order to engage students for active learning and creativity. In addition, infographics work to match the visual representations in mathematical communication and connection (Alshehri & Ebaid, 2016). Infographics could be used as visual learning tools which highly effective designs for mathematical communication skills by preparing the following steps (Davis & Quinn, 2014): (1) Determining the objectives (2) Determining the type of infographics (3) Presenting the data in clear a way, and (4) Deciding the components of infographics.

Preparation the use of infographics for 'Linear Programming', the researchers determined a specific objective--'to simplify the mathematics concepts and processes to solve linear programming problems using graphical method'. The types of infographics presenting information using 'Informational infographic' (Figure 1) and 'Process infographic' (Figure 2).

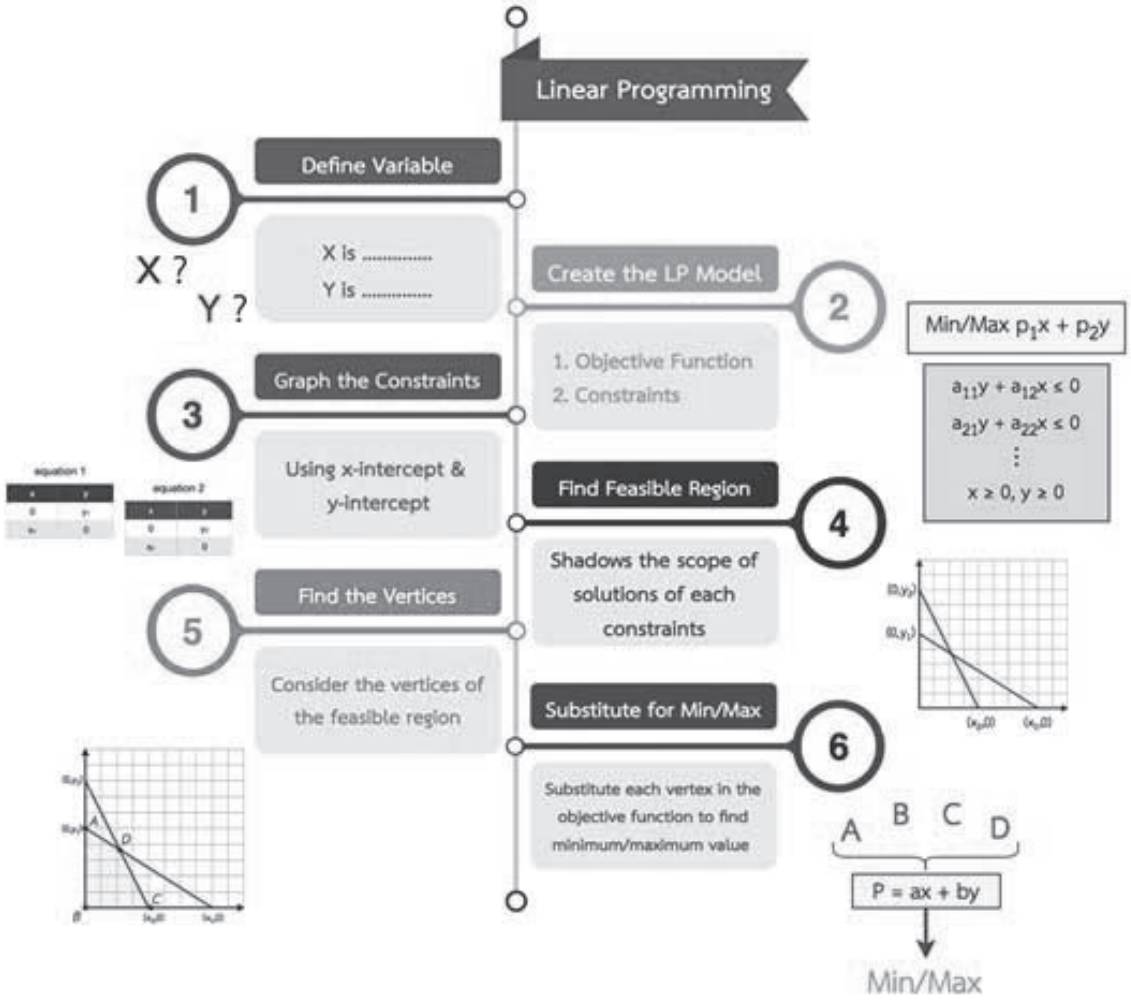

Figure 1 Graphing linear equation and inequalities

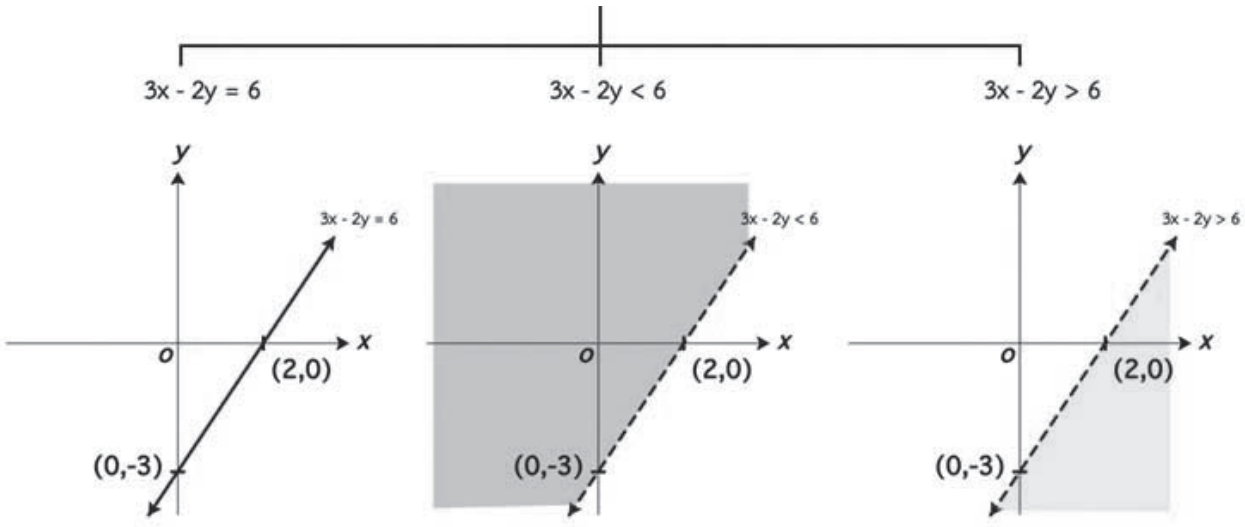

Figure 2 Steps for solving linear programing

Related Works

McGowen & Tall (2010) introduced the notion of met-before and mentioned about how new learning is affected by the learner's previous experiences and these experiences can support new learning. In addition, a belief of miscommunication (i.e., the minus symbol to indicate subtraction) becomes difficulties for many students in subsequent topics in mathematics. They studied the problematic nature of prior arithmetic thinking of 128 college students, a majority of students had misconceptions on given the unknown numbers, for examples, '2y is larger than y because it has a number in front of the variable' as marked on the number line and interpret the minus sign in front of the number -32 = (-3)(-3) = 9.

Results

The collection data were analyzed to support the investigation of students' previous knowledge in terms of misconceptions in solving linear equations and inequalities as shown in Table 2 which displays the students' misconception based on the quiz. Therefore, the number of misconception responses in each topic was counted and calculate the percent of responses to population.

Singh & Jain (2017) conducted the research aimed to identify the dyscalculic students with difficulty in processing and memorizing sequences in mathematical learning. They found that after giving instruction with infographics design can support dyscalculic students' image processing ability and achievement motivation better than traditional design. In addition, dyscalculic students focus on images, diagrams and figures to solve the mathematical problems by their understanding ability and learn mathematical concepts easily and for long time.

Methods

The purpose of this study was to analyze students' misconceptions and errors in mathematical process skills. The researchers developed a set of quizzes to test 31 Grade 12 students on relevant topics (solving linear equations and inequalities) then investigated their prior knowledge, misconceptions and errors in mathematical process skills.

Table 2 : The students' misconceptions in linear equations and inequalities

Topic	Descriptions of Misconceptions	Misconception Responses by Students (N = 31)
Solving of linear equations by substitution method	- Confusing about solving for one of variables in terms of others; - Fail to substitute the expression 'variable = ...'into the remaining equation; - Improperly applying mathematical operations to combine like or non-like terms.	5 (16.13%)
Solving of linear equations by elimination method	- Uses the wrong operation (+, -, ×, ÷) when eliminating a variable term from an equation; - Applies the wrong rules to eliminate one of the variables; - Combines like term incorrectly.	9 (29.03%)
Solving of linear equations by graphing method	- Has difficulty with points (ordered pairs) on the axes; - Fails to find the ordered pairs of x-intercept and y-intercept satisfy the given equations; - Fails to realize that all ordered pairs on the graph satisfy the given equations; - Has difficulty determining the solution to the linear system with the intersection point.	15 (48.39%)

Table 2 : The students' misconceptions in linear equations and inequalities (cont.)

Topic	Descriptions of Misconceptions	Misconception Responses by Students (N = 31)
Solving inequalities	- Fails in multiplying or dividing each side of inequality by a negative quantity; - Has difficulty interpreting the results of inequalities.	17 (54.84%)
Graphing inequalities	- Incorrectly identify the test-point to determine the solution region; - Fails to graph boundary lines and shade solution region.	18 (58.06%)

In addition, the examples of analyzing students' misconceptions and errors in mathematical process skills (did not indicate clearly in A, B, C and H or did not indicate scales and equations on the graph in F and D which cause wrong graphing of solution in G or did not extend the line in E) were shown in the Figure 3 and 4.

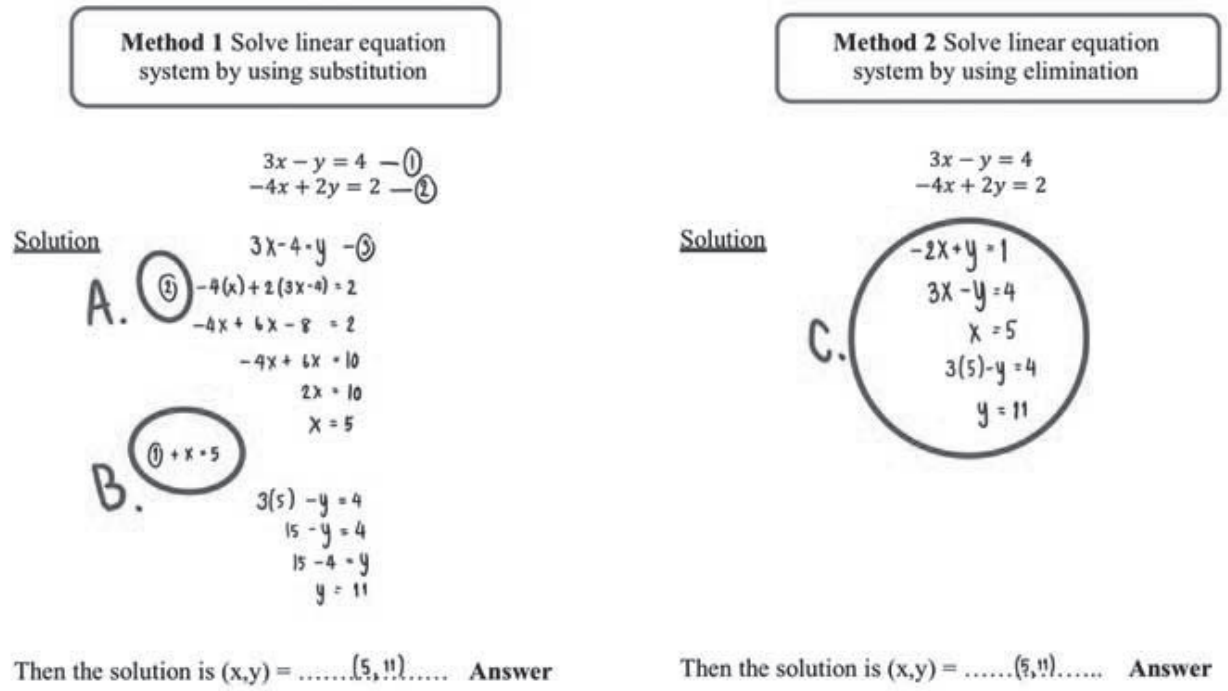

Figure 3 Simulation of students' misconceptions I

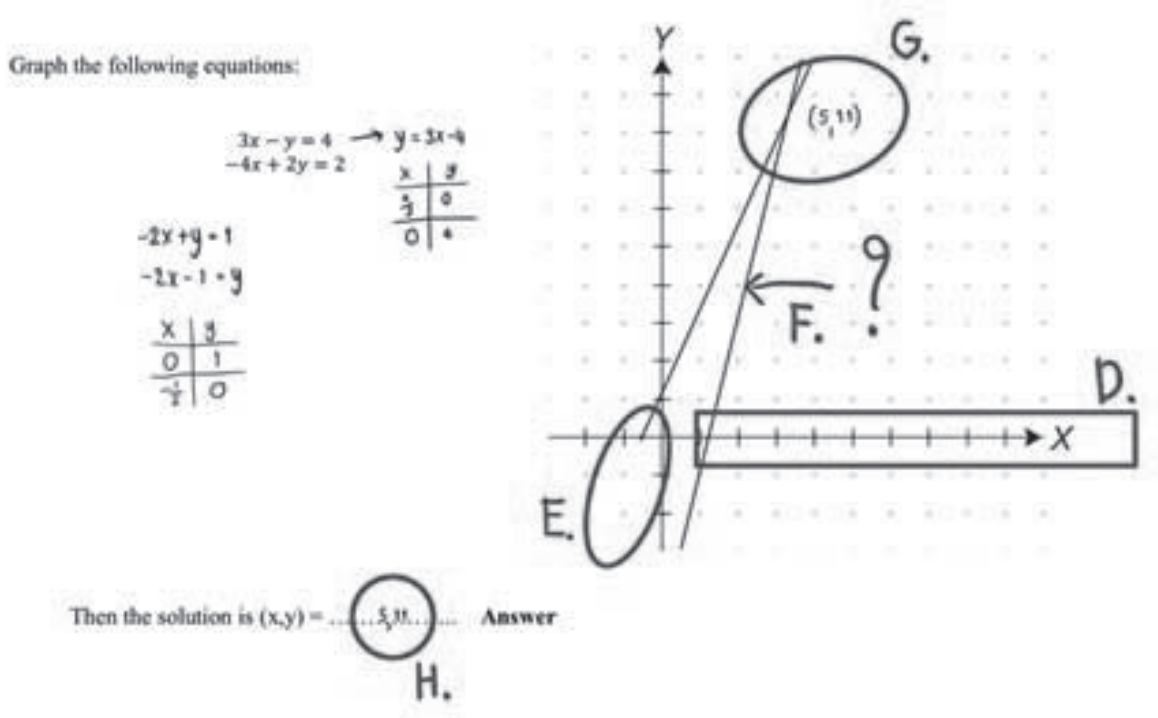

Figure 4 Simulation of students' misconceptions II

Conclusion and Future Work

This study was aimed to investigate students' previous misconceptions and errors in mathematical process skills from a set of quizzes before applying the real classroom action research. According to the data analyses in this study, the students' prior misconceptions and errors are found as the followings :

(1) Misconceptions in algebraically solve given linear equations with integral coefficients by using substitution and elimination methods;

(2) Misconceptions in algebraically solve given linear inequality by using properties of inequality;

(3) Misconceptions in graphically solve given linear inequality using;

(4) Indicated each step of algorithm incorrectly or did not plot graphs clearly.

After conducting this study, the researcher plan to design new strategies to correct students' misconceptions in learning linear system including linear equation and inequalities. The next research should be done to improve student's mathematical communication skills by using infographics (information + graphics) in learning 'Linear Programming'. The infographics strategy previously mentioned is appropriate to match the visual representations in mathematical communication and connection (Alshehri & Ebaid, 2016).

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